

Solution 7

Supplementary Problems

1. Let f be a function on $[a, b]$. Verify that the parametric curve $x \mapsto x\mathbf{i} + f(x)\mathbf{j}$ is regular provided f is continuously differentiable on (a, b) .

Solution. Let the curve be $\mathbf{c}(x) = x\mathbf{i} + f(x)\mathbf{j}$. We have $\mathbf{c}'(t) = \mathbf{i} + f'(x)\mathbf{j}$ and

$$|\mathbf{c}'(t)| = \sqrt{1 + (f'(x))^2} > 0 ,$$

hence \mathbf{c} is regular.

2. Let \mathbf{c} be a regular parametric curve on $[a, b]$. Find a parametric curve γ whose image is the same as \mathbf{c} but reverse the orientation.

Solution. Define

$$\gamma(t) = \mathbf{c}(a + b - t) \quad t \in [a, b] .$$

Then $\gamma(a) = \mathbf{c}(b)$ and $\gamma(b) = \mathbf{c}(a)$. Moreover, $\gamma'(t) = -\mathbf{c}'(a + b - t)$ so $|\gamma'(t)| = |\mathbf{c}'(a + b - t)| > 0$, γ is a regular parametric curve.